Student Name: ______________________________

Student Number: ____________________________

GE 213.3 - Mechanics of Materials

MIDTERM EXAMINATION

March 3, 2004

Time Allowed: 2 Hours | Professor: B. Sparling

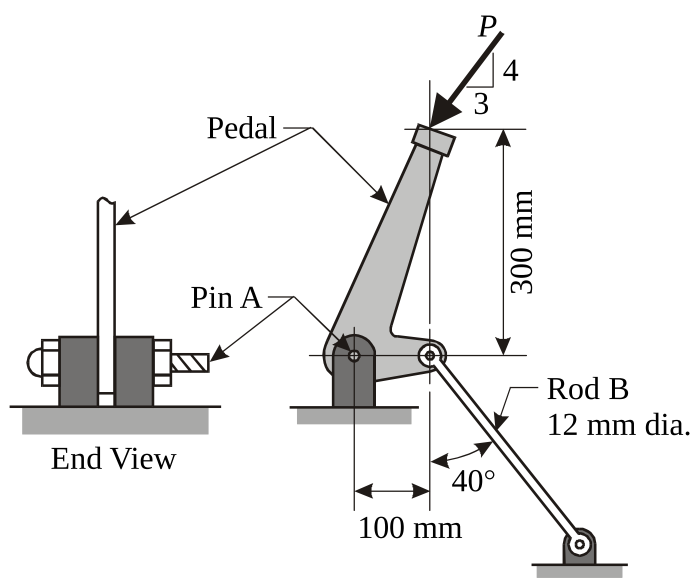
Notes:
- Closed book examination; Calculators may be used
- The value of each question is provided along the left margin
- Supplemental material is provided at the end of the exam (i.e. formulas)
- Show **all** your work, including all formulas, calculations and **units**
- Write your work in the space provided on the examination sheet.
  (The backs of the examination sheets may also be used if required)

MARKS

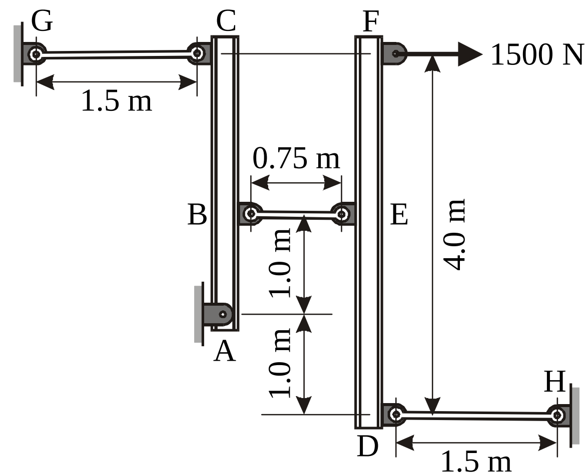
20 **QUESTION 1:** A control pedal is acted on by the inclined force *P*, as shown below.

a) Determine the maximum permissible magnitude of force *P* based on an allowable normal stress of 90 MPa in Rod B.

b) Determine the corresponding size (diameter) required for Pin A if the allowable shear stress in the pin is 60 MPa.



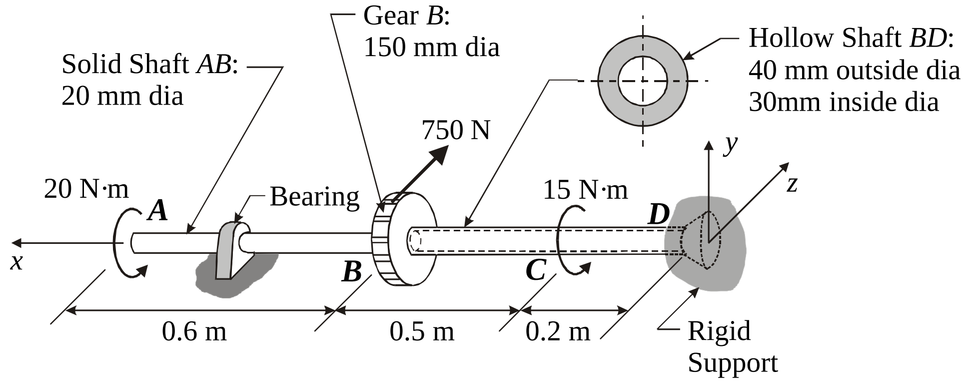
21 **QUESTION 2:** Two perfectly rigid vertical beams, ABC and DEF, are connected to each other and to supports at Points G and H by three horizontal rods (CG, BE and DH), each with a cross-sectional area of 30 mm^2 and an elastic modulus of 200 GPa. If a horizontal force of 1,500 N is applied at Point F as shown, determine the resulting horizontal displacement at Point F.



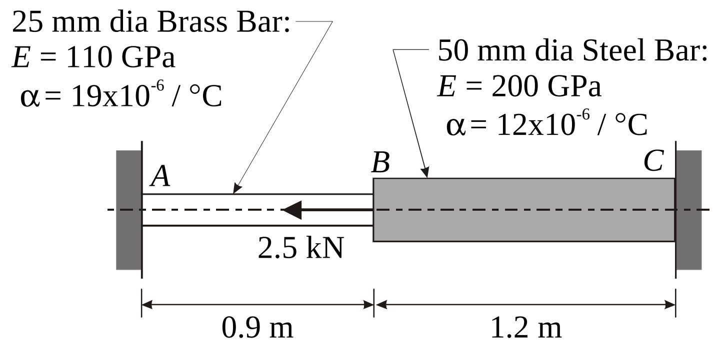
22 **QUESTION 3:** A shaft assembly features a solid shaft segment *AB*, a gear at *B*, and a hollow shaft segment *BD*, as shown below. Shaft segment *AB* is supported by a frictionless bearing, while shaft segment *BD* is rigidly connected to a fixed support at end *D*. In addition to the applied torques at Points *A* and *C*, a 750 N force is acting parallel to the z-axis at the outer surface of the gear at *B*. Both shafts are made of steel with $G = 77$ GPa.

a) Determine the maximum torsional shear stress in shaft segment *BC*.

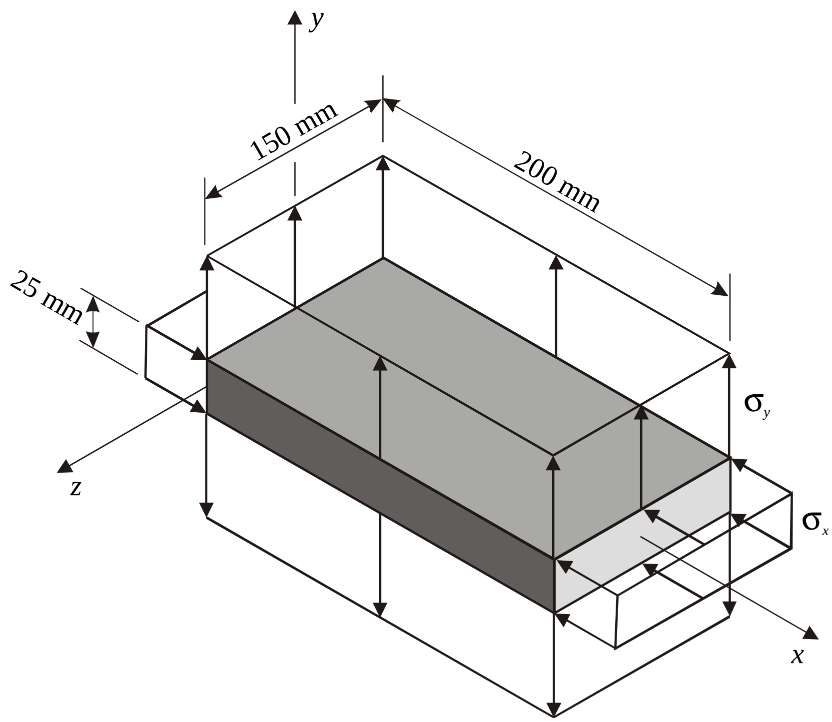
b) Determine the angle of twist at Point *A*.



22 **QUESTION 4:** At a temperature of 20 °C, prior to the application of the 2.5 kN force, the assembly made up of a 25 mm dia brass bar and a 50 mm dia steel bar fits snugly between the frictionless, rigid supports at *A* and *C*; at this stage, both bars are completely unstressed. If the temperature of the assembly is then increased to 40 °C and the 2.5 kN force is applied as shown below, determine the resulting axial force in the brass bar (*AB*).



15 **QUESTION 5:** A titanium plate, with material properties of E = 120 GPa and ν = 0.35, has the initial dimensions indicated below when it is completely unstressed. The plate is then subjected to uniform normal stresses in the x and y directions (σ_x and σ_y) as shown. The total resultant force acting on one surface in the x direction is 100 kN, while the total force acting on one surface in the y direction is 250 kN. Estimate the final (stressed) dimensions of the plate for this loading condition.



Supplemental Material:

- **Static Equilibrium:** $\Sigma F_x = 0 \,;\, \Sigma F_y = 0 \,;\, \Sigma F_z = 0 \quad \& \quad \Sigma M_x = 0 \,;\, \Sigma M_y = 0 \,;\, \Sigma M_z = 0$
- **Normal Stress:** $\sigma_{avg} = \dfrac{P}{A} \qquad F = \int_A \sigma \,\mathrm{d}A$
- **Direct Shear Stress:** $\tau_{avg} = \dfrac{V}{A}$ (Single) or $\tau_{avg} = \dfrac{V}{2A}$ (Double)
- **Bearing Stress:** $\sigma_b = \dfrac{P}{t\,d}$
- **Allowable Stress:** $F.S. = \dfrac{P_U}{P_D}$ or $F.S. = \dfrac{\sigma_U}{\sigma_D}$ **;** $\sigma_{all} = \dfrac{\sigma_U}{F.S.} \qquad P_{all} = \sigma_{all}\, A \qquad A_{req} = \dfrac{P_D}{\sigma_{all}}$
- **Stresses on Oblique Planes:** $\sigma_\theta = \dfrac{P\cos\theta}{A_o/\cos\theta} = \dfrac{P}{A_o}\cos^2\theta \,;\quad \tau_\theta = \dfrac{P\sin\theta}{A_o/\cos\theta} = \dfrac{P}{A_o}\sin\theta\cos\theta$
- **Average Normal Strain:** $\varepsilon = \dfrac{\delta}{L_o} = \dfrac{L^* - L}{L}$
- **Hooke's Law:** $\sigma = E\,\varepsilon$
- **Axial Deformations:** $\delta = \dfrac{P\,L_o}{A_o\,E}\,; \qquad \delta_{tot} = \sum_i \dfrac{P_i\,L_i}{A_i\,E_i}\,; \qquad \delta = \int_0^L \dfrac{P(x)}{A(x)\,E(x)}\,\mathrm{d}x$
- **Thermal Deformations:** $\delta_T = \alpha\,(\Delta T)\,L_o \qquad \varepsilon_T = \dfrac{\delta_T}{L_o}$
- **Poisson's Ratio:** $\varepsilon_y = \varepsilon_z = -\nu\,\varepsilon_x \qquad \varepsilon_y = \varepsilon_z = -\dfrac{\nu\,\sigma_x}{E}$
- **General Hooke's Law:** $\varepsilon_x = \dfrac{\sigma_x}{E} - \nu\dfrac{\sigma_y}{E} - \nu\dfrac{\sigma_z}{E}\,;\ \varepsilon_y = -\nu\dfrac{\sigma_x}{E} + \dfrac{\sigma_y}{E} - \nu\dfrac{\sigma_z}{E}\,;\ \varepsilon_z = -\nu\dfrac{\sigma_x}{E} - \nu\dfrac{\sigma_y}{E} + \dfrac{\sigma_z}{E}$
- **Shearing Strain & Stress:** $\theta^* = \dfrac{\pi}{2} - \gamma_{xy}\,; \qquad \gamma_{xy} = \dfrac{\tau_{xy}}{G}\,; \quad \gamma_{yz} = \dfrac{\tau_{yz}}{G}\,; \quad \gamma_{zx} = \dfrac{\tau_{zx}}{G}\,; \qquad G = \dfrac{E}{2\,(1+\nu)}$
- **Resultant Torque:** $T = \int_A \rho\,\tau\,\mathrm{d}A$
- **Torsional Strains:** $\gamma = \dfrac{\rho\,\phi}{L} \qquad \gamma_{\max} = \dfrac{c\,\phi}{L} \qquad \gamma = \left(\dfrac{\rho}{c}\right)\gamma_{\max}$
- **Torsional Stresses:** $\tau = \left(\dfrac{\rho}{c}\right)\tau_{\max} \qquad \tau_{\max} = \dfrac{T\,c}{J} \qquad \tau = \dfrac{T\,\rho}{J} \qquad J = \int_A \rho^2\,\mathrm{d}A = \dfrac{\pi}{2}c^4$
- **Torsional Angle of Twist:** $\phi = \dfrac{T\,L}{J\,G}$
- **Torsion - Gear Compatibility:** $\phi_1\,\rho_1 = \phi_1\,\rho_2$
- **Pure Bending - Normal Strain:** $\varepsilon_x = -\dfrac{y}{\rho} \qquad \varepsilon_{\max} = c/\rho \qquad \varepsilon_x = -\dfrac{y}{c}\varepsilon_m$
- **Pure Bending - Normal Stress:** $\sigma_x = -\dfrac{y}{c}\sigma_m \qquad \sigma_x(y) = -\dfrac{M\,y}{I} \qquad \sigma_{\max} = \dfrac{M\,c}{I}$
- **Bending – Section Properties:** $I = \int y^2\,\mathrm{d}A\,;$ Centroid: $\int y\,\mathrm{d}A = 0$